

Preface

Leibniz's thought may first appear as a *fantastic fairy tale* or an enthralling and imaginative *metaphysischer Roman*. Profound, indeed, and visionary, and endowed with all the traits of genius; but still confused, fragmentary, abstruse, arbitrary. Such disconcertment at a first reading is so very common, and witnessed by the authority of so many interpreters, to have become commonplace in philosophical criticism. Meaning to offer more than a simply historical account, however, this same philosophical criticism endeavors to show that, at least at second sight, Leibniz's own desultory and fantastic exposition of metaphysics lends itself to arrangement into a consistent, perhaps verisimilar, and in any case not airy-fairy framework. Participating in such a critical endeavor, the present book also aims at offering a reasonable account of Leibniz's thought on space as it appears in the last years of the philosopher's life – as complete and consistent an account as possible.

It is not difficult to determine how many obstacles may prevent this exposition from being a complete one. Some of them are objective, and arise from the fact that Leibniz's theory of space is itself incomplete in many respects. The most important document of Leibniz's mature spatial doctrine that has been handed down to us, his correspondence with the Newtonian Samuel Clarke, goes back to 1716, the very last year of Leibniz's life; and, apparently, it was the occasion of such a correspondence that

prompted most of Leibniz's reflections on the subject – reflections that were terminated, just as the famous exchange was, with Leibniz's sudden death. Then, many concepts still needed to be refined and, having been only little or not at all investigated by Leibniz's ingenious mind, many other issues that would be necessary to a comprehensive metaphysics of space now can only be laboriously reconstructed or integrally conjectured. These quite relevant objective obstacles are matched by subjective ones. Conspicuous editing gaps still affect Leibniz's corpus of works. Its critical edition proceeds according to chronological criteria that have hitherto permitted the publication of Leibniz's philosophical writings only to 1690 and his mathematical writings to 1676 (even though an important collection of geometrical writings dating from 1679 is available). But not before the turn of the century did Leibniz begin to develop his spatial metaphysics, and only between 1712 and 1716 did he devote his best efforts to it. Therefore, one has no choice but to resort to nineteenth-century editions, which offer a few anthological selections from Leibniz's late manuscripts – intelligent selections, most of the time, but still not the whole thing. Other difficulties pertain to Leibniz's systematic statement, or lack thereof. In fact, even our well-meaning “second sight” on Leibniz's thought, by which we would want to stress the inherent necessity of his doctrine beyond the occasional and fragmented account he has himself given of it, immediately clashes against the opposite problem. The more we read Leibniz, that is, the more we are faced with a congeries of systematic connections between his diverse theories, and it is only with difficulty that we can isolate a single argument from the whole of Leibniz's philosophy. As soon as we lose sight of some (however tenuous) links between a particular argument and the other ones, it will appear just as unfounded, inconsistent, or fantastically arbitrary as the fairy tale or the romance. Hence, the present account of Leibniz's philosophy of space will also need, from time to time, to venture uneasily into domains that are apparently very distant from it, but however so intimately connected to Leibniz's metaphysical framework that it is impossible not to get a little entangled with them. On the other hand, the risk of extravagance inherent in some such digressions seems to me certainly to be preferred to the risk incurred by some other accounts of Leibniz's spatial philosophy that, overlooking whatever goes beyond Leibniz's letters to Clarke, end up by simply retelling the old strange story.

Pursuing thus a comprehensive and systematic picture, we arrive at the distinctive trait of this essay. Let us first acknowledge that a great many

accounts of Leibniz's philosophy of space have indeed been accurate, and keen to render its overall spirit. Covering all stages of Leibniz's production, they have also attentively considered the logical premisses of such a metaphysical purview, its theological consequences, its dynamic implications, and much more. However, as it happens, all or almost all of them have failed sufficiently to investigate what remains, in my opinion, the greatest contribution Leibniz has ever given to his theory of space in non-strictly philosophical terms – his work on geometry.

One must consider, in fact, that ever since his early mathematical studies Leibniz took it upon himself to found a new geometrical science, that he called *analysis situs*. Owing to its widespread applications, powerful instruments, simple uses, and yet other advantages, in Leibniz's expectations this new discipline was by far to surpass Euclid's classical geometry, which was much studied in the seventeenth century. The *analysis situs*, or Analysis of Situation, has been a very obscure chapter in the history of Leibniz's critical interpretation. Even from the strictly mathematical point of view, it has received no adequate treatment. However often it is mentioned (with such an awe that borders on indifference), it has remained almost impenetrable to the understanding of the interpreters. They have continued to evoke the ghosts of modern geometrical disciplines (such as vector calculus, projective geometry, or topology), thus attempting to classify a science that has little to do with anything of the above – and that, in any case, needs more to be studied than pigeonholed. Further on, we will be better able to understand the reasons for such a poor critical reception. To be sure, they are most to be found in the small number of published texts on this discipline, as well as in today's somehow attitude of specialization, which sees the history of philosophy and the history of mathematics on two different tracks, and which is of course totally extraneous not only to Leibniz but also to the seventeenth-century spirit in general. Here, we need to emphasize that his studies on *analysis situs* were not an isolated or marginal episode in his overall research. So much is witnessed by his constantly pursuing them (almost uninterruptedly from 1679 to his death), as well as by the number of (by now, also published) letters in which he tells his friends about his results, and finally, by the large quantity of unpublished Leibnizean manuscripts to be found in the *Leibniz-Archiv* in Hannover which deal with the subject from every point of view. In short, the studies on *analysis situs* occupied Leibniz's mind no less than his research on the *characteristica universalis* did, and they had no lesser impact on his metaphysics.

It is actually another cliché of philosophical criticism that studying Leibniz's mathematics will certainly help the understanding of Leibniz's philosophy. This piece of wisdom is however only followed by most interpreters in the narrow sense that confines Leibniz's mathematics to his most important and celebrated discovery, namely infinitesimal analysis – however, recently a few studies on combinatorial analysis and probability theory have begun to appear. But the Calculus is certainly the least suitable domain to chose for studying the *objective* encounter between mathematics and philosophy in Leibniz. If we look specifically at the references that, in his metaphysical writings, Leibniz frequently makes to his own studies on infinitesimal analysis (for example, when he compares contingent truths to asymptotic curves), we will easily see that, for him, infinitesimal analysis must have simply played the important but subjective role of being a formidable heuristic instrument that, through a thick net of (sometimes misleading) analogies, could help him with some of his most famous and creative philosophical theories. This picture dramatically changes when on the contrary we look at the writings on situational analysis that go back to the last period of Leibniz's production. Clearly, here the geometrical studies, which in Leibniz's young years had gone along with his logical and epistemological studies, become indispensable metaphysical instruments for objectively determining space. By his *analysis situs*, Leibniz argues he can demonstrate the continuity of space, its tridimensionality, the possibility of rigid motion in it, its Euclidean nature, or its absolute necessity. He even relies on geometry in founding some of the most delicate passages of his phenomenalism. That space is actually constituted by points, though here abstractly meant as terms of situational relations, is perhaps the highest result of Leibniz's geometrical investigation and, at the same time, it also marks the core of Leibniz's theory of phenomenal expression. It shows in fact that a set of non-spatial relations (such as those occurring between monads) can be isomorphic to ("expressed by") a set of situational relations that per se suffice to produce phenomenal extension and thus, ultimately, faithfully represent the supersensible through the sensible. So much so, that one may hold that Leibniz's whole phenomenal theory finally stands on these two very concepts, isomorphism and situation – which both are taken from geometry. Furthermore, Leibniz's so-called spatial relationism (or relativism) – which has been the object of numerous studies and disordered idealizations in the dynamic and philosophical spheres (finding there as many meanings as its interpreters) – may well have its primary

origin and meaning in the geometrical theory of situation. Thus, Leibniz's geometrical studies are not analogies or tools in the art of metaphysical discovery as much as the continuation of metaphysics itself by other means.

Good metaphysics, of course, as well as bad metaphysics. In much the same way in which such geometrical writings need to be taken into consideration in order for us to gain a further inch of sense out of Leibniz's metaphysical romance, they also prove useful in clearing up some minor or major difficulties that, either in a transitory or definitive way, may hinder all attempt at reconstructing Leibniz's spatial philosophy. For example, the most serious setback incurred by Leibniz's theory of matter and phenomenon seems to me to reside in its determination of the boundary of an organic body, which basically rests on the Aristotelian notion of contiguity. Faulted at its very root, and hardly emendable as it is, such a notion can have no use in good reasoning. If we apply ourselves to following Leibniz's geometrical progress over the years, we will see that the concept of contiguity, never earnestly criticized in philosophical terms, has been laboriously discussed again and again, demolished and rebuilt anew throughout dozens of essays on the analysis of situation. Since it has never been radically erased, it eventually pollutes Leibniz's metaphysical argument. It is however more a geometrical mistake than a philosophical one. By the same token, the second serious fault in Leibniz's theory of expression (which only later will we be able to expound clearly, as it needs a few technicalities) resides in Leibniz's failing to demonstrate the characterization of quality (phenomenologically defined through a coperceptual act) by means of the angular geometric relation on which Euclidean similarity rests. This lack of demonstration – a most serious one in foundational terms, and one that risks destroying the bridge between situational analysis and metaphysics – also is an essentially mathematical error deriving from Leibniz's insufficient consideration of transformation groups. Of course, we cannot hold Leibniz responsible for not having offered an adequate definition of continuity (on the contrary, we will marvel at how close he has come to that, and thus how greatly he has exceeded all geometers of his century and the following one), nor for having ignored Klein's theory. The fact remains that some basic metaphysical obstacles hindering Leibniz's mature monadological doctrine can only be understood through a rigorous study of the geometrical thought that underlies it.

Proceeding towards (ideal) systematic completeness in my exposition of Leibniz's philosophy of space, addressed to the best possible comprehen-

sion of it in theoretical terms, my main purpose has been therefore that of conquering the vast region of the analysis of situation to the studies on Leibniz's metaphysics. Since research into this obscure geometrical science does not abound, and on the contrary much material still needs to be sorted out, I have also obtained a (perhaps still rough) first exposition of Leibniz's *analysis situs*. On this specific topic, to be sure, I have pursued no ideal of completeness – as it would be frustrated in any case, at least until many other unpublished texts find their way into print. The selection I have made among Leibniz's geometrical texts corresponds to two main criteria. The first one is chronological. I have, that is, favored and expounded only the theories dating back to the last period of Leibniz's mathematical work. The second criterion is a theoretical one. In the impressive quantity of issues discussed by Leibniz, I have in fact chosen those having a more relevant interest in foundational terms and a more evident metaphysical influence. As it has already been mentioned, both criteria are arbitrary. The first, because there are studies on the analysis of situation dating more or less in any year of Leibniz's life; the second, because most of these studies either digress to disciplines other than metaphysics (particularly, to studies on the universal characteristic), or get entangled in exquisitely geometrical topics, and there they remain. On the other hand, these two criteria have nonetheless the advantage that they almost perfectly overlap, because it is almost only in the last period of his life that Leibniz was actively concerned with spatial metaphysics, and therefore it is his geometrical work from those years that most keeps the signs of a philosophical symbiosis.

The general outline of my study is as follows. It is divided into four chapters, two of which are devoted to the *analysis situs* from the point of view of geometry, and two to a general interpretation of Leibniz's spatial metaphysics. Chapter 1 offers a brief survey of Leibniz's geometrical research, meant to elucidate the development, and thus the interest, of Leibniz's geometrical results from the period 1712–16 which most of the following investigation will deal with. In any case, serving as an overall perspective on the analysis of situation, it will discuss the main sources that might have influenced Leibniz in the making of his new discipline, as well as the main episodes of the fortune of the *analysis situs* following the death of its creator. Chapter 2 attempts to reconstruct, in modern mathematical terms, the main results of Leibniz's analysis of situation, and chiefly those most relevant from the philosophical point of view. I have tried to limit the formal apparatus to a bare minimum. Chapter 3 attempts to show how

situational analysis is a requisite for Leibniz's theory of expression, which constitutes the heart of his late monadological metaphysics. In a most general and abstract way, it discusses the relation between monads and the phenomenal world, as well as some central points of Leibniz's theory of knowledge, and chiefly the constitution of sensibility. In conclusion, it arrives at a general definition of space. Chapter 4, finally, attempts more concretely to found Leibniz's theory of extension. It will show how it is from his general definition of space and his theory of expression that Leibniz deduces the many determinations he attributes to physical space and the ideal space of geometry. A few basic concepts will be introduced here (such as those of matter, perfection, and corporeal substance) that are actually the main features appearing in Leibniz's correspondence with Clarke and in all the interpretive readings of Leibniz's theory of space. These four chapters alternate with three interchapters, or Additions, that deal with somehow marginal or technical issues having nonetheless some relevance to our general account. In the Appendix, finally, I transcribe some unpublished Leibnizean manuscripts on the analysis of situation. Most of them are fragments going back to various dates, which I have deemed especially meaningful for the metaphysical developments of Leibniz's geometry. But I have also transcribed six longer essays from the last period of Leibniz's production that, along with another three already published essays, are likely to constitute the most important (and perhaps the only) findings of the late Leibniz's inquiry into the *analysis situs*.

As they are meant to offer the first account ever of a discipline ignored by most interpreters thus far, Chapters 1 and 2 have quite naturally lent themselves to retrospective examinations of Leibniz's geometry. Thus, they abound more in references to authors and theories prior to Leibniz and likely to have inspired him, than in references to future developments of his discipline (which, as we will see, enjoyed a fortune that only with generosity may we term as scarce). On the contrary, Chapters 3 and 4 on metaphysics have been written from a forward-looking perspective, and thus they often refer to authors posterior to Leibniz. There have been several reasons for such a choice. First, quite a number of historical reconstructions of Leibniz's philosophy already exist, so that there seemed to be no point in coming up with one more. Second, we ought to keep in mind that in the period 1712–16 Leibniz's ideas on space were so very personal and strictly linked to his original monadological framework that there would not be much sense in contrasting them with those of, say, Descartes or Hobbes.

By that time, in fact, Leibniz's thinking was the result of a fifty-year-long elaboration of those ideas he had learned about in his prime. The study of them can therefore only prove useful if we confine ourselves to the consideration of Leibniz's early metaphysical efforts. Finally, given the enormous interpretive problems that torment such a fragmentary and incomplete philosophy of space as that of the late Leibniz, it may not be wrong to try and reconstruct some of its gaps by relying on the inferences made by, say, a brilliant and accurate interpreter shortly following in time – and thus, much closer to the source than we are. Such a hermeneutic criterion may at least prove useful in understanding the possible outcomes of the theory of space that Leibniz was developing in the years immediately preceding his death and that he left unfinished.

If this is true, then, the first reference figure that comes to mind is undoubtedly Kant. In many respects, Kant was the heir of Leibniz's best philosophical intuitions. He was also the celebrated author of a complex and controversial theory of space that he understood as quite opposite to the Leibnizean one. However this may be, the general tendency of the last period of Leibniz's metaphysics seems to me clearly to head towards a transcendentalistic outcome. This point will actually be a central one in my interpretation. In this sense, yet another cliché of philosophical criticism, according to which Wolff and his disciples have totally misunderstood Leibniz's philosophy, and thus represented more an obstacle than a bridge between Leibniz's genius and Kant's, seems to me, at least if confined to spatial metaphysics, to be true. In Chapters 3 and 4, I will in fact attempt to show, however incidentally and in very general terms, how Wolffian metaphysics has lost almost all traces of Leibniz's original ideas on the nature of space. On the contrary, I will also argue, Kant's attempts, not only in his pre-Critical period but even in most works of his Critical period (from the Amphiboly of the *Critique of Pure Reason* to the Transcendental Antinomy), have been actually meant to retrieve some original characteristics of Leibniz's metaphysics that had been lost in the preceding fifty years of other studies and erroneous interpretations. On the other hand, I would definitely not join the army of those Kantian scholars who view Leibniz as the genuine forerunner of Critical philosophy. Nor would I support the bizarre view according to which he would have anticipated Boole and Frege, Kant and Einstein, topology and non-standard analysis, and in sum he would always be an *in nuce* halved genius. Rather, I would think it helpful for the comprehension of Leibniz's metaphysics to compare and

contrast it with the better-known Kantian doctrine of the pure forms of intuitions. In this way, it will be clear how the transcendentalistic attempts of Leibniz's late writings on *analysis situs* have steadily pointed in another direction from the one that philosophy was to take in Königsberg – a better direction in some respects, and a worse one in others. In short, I would argue, transcendentalism, as early as the beginning of the eighteenth-century, did have a non-Kantian variant. Just to put both the difference and the similarity between Leibniz's and Kant's philosophy of space in a formula, I would suggest that for Leibniz space is indeed a form of intuition, but it is not a formal intuition. That it is a form of intuition replicates (just to mention what first meets the eye) the distinction between phenomena and noumena, as well as the possibility of a transcendental determination of a phenomenon. That space is not a pure intuition immediately implies, on the other hand, that things can be known in themselves, that space itself can be fully determined by the understanding, and that geometry is a perfectly analytic science.

The forward-looking perspective of Chapters 3 and 4, however, stops at Kant. After him, in fact, no thinker seems to me to have effectively discussed Leibniz's metaphysics as a still active inspiration in cultural terms. Even post-Kantian German Idealism, though often praising single features of Leibniz's doctrine (from its spiritualism to its vitalism), was no longer interested in Leibniz's philosophy as a whole, which it already regarded, in effect, just as the *metaphysischer Roman*. All the following revivals of Leibniz's thought, from formal logic to Husserl's monads, and beyond, have been local interpretations or antiquarian suggestions. For all of them, monadology irretrievably belongs to the past, and is no longer an enemy to fight, nor an ideal to pursue.

At this point however I cannot avoid saying a few words on the interpretive criteria I have employed and the historical consistency of my account of Leibniz's spatial metaphysics. From the point of view of its exposition, I have favored as wide a liberty as possible. In Chapter 2, in which Leibniz's main geometrical results are discussed, I have had no qualms about using lexicon and concepts from contemporary geometry, which alone allow us fully to understand the real successes and the actual limits of Leibniz's endeavor. Thus, even though Leibniz could hardly be expected to know about Riemannian manifolds, non-Euclidean geometries, isometry groups, or even sets, it seems to me that the only sensible and useful reconstruction of Leibniz's geometry cannot but pass through a (however

strained) reformulation of its problems in today's terms. As for the philosophical lexicon, I have mostly favored Kantian terminology (of course, only when the Leibnizean one proves insufficient), according to the above indications on the specific historical perspective that has been applied. A further advantage of this choice is that Kant's system of definitions is still widely comprehensible. In philosophy, thus, I have only rarely resorted to a more modern terminology. The only exception concerns perhaps the concept of "intentionality" that, however possibly familiar to Leibniz from middle-age sources, is assumed here (a bit generically, as it suffices in this context) in its phenomenological usage. In any case, most of the time the present account does not proceed textually (because all of Leibniz's writings would be too fragmentary for the purpose). Instead, it proceeds according to a very elaborate, systematic, and abstract order, mentioning the original Leibnizean passages to which it refers in the notes (or at times, in translation, in the text). As a result, some celebrated Leibnizean propositions (such as the non-existence of a vacuum and the impenetrability of matter, or the Principle of Indiscernibles, or the fact that space is not the *sensorium Dei*) have been demonstrated by a much shorter proceeding than the one Leibniz himself made use of in his (mostly, polemical) writings – in which, of course, Leibniz could have never taken the overall framework underlying his isolated statements for granted. In any case, I have always tried to provide an explanation for each such variation, so that the reader may understand its systematic connection and, at the same time, not to lose sight of its historical occasion.

Now, I will come to consistency, which, along with completeness, was meant to be a criterion of my interpretation. Such non-contradictoriness is not everywhere to be found in Leibniz's texts. Even limitedly to the last period of Leibniz's production, what we have is a number of short essays scattered over the years. As their author's thought constantly evolves, these essays are not always consistent with one another. Leibniz made most of his discoveries in the very last years of his life, and he did not always realize how greatly they conflicted with the old views that he continued to hold. Most of the writings that we will take into consideration are furthermore private notes not meant for publication, in which sometimes Leibniz tried paths that he would soon abandon. It would be ungenerous indeed to expect absolute rigor from outlines, first drafts, and tentative notes. On the contrary, we should take a liberal attitude, I believe, in judging an author who used to say that it is better to take whatever good there may

be in a book rather than wasting time in criticizing its defects. Whenever two loci are so different as to be incompatible, I have chosen the one that seems more suitable to the general framework, all the while signaling, of course, Leibniz's ambivalence on that particular point. Not blameless himself, in a few cases Leibniz shrewdly adopts the same strategy. Thus, for example, when phenomenalism leads him to distinguish between an objective phenomenon and a subjective one, in his disputes he gets away with it by assuming now one and now the other possible meaning of a phenomenon. (See the following: Matter as an objective phenomenon is actually subdivided to infinity – and Leibniz admits it; however, matter as an objective phenomenon should be also in an absolute inertial reference – and Leibniz does not admit it, because, he says, the objectivity of matter is only an ideal one; and there are other similar instances.) When in a text Leibniz's argument lacks a passage, I have tried to reconstruct it. When on the contrary a whole part of his general theory is missing, I have just acknowledged it. The only somewhat arbitrary construction I have attempted almost *ab ovo* concerns a few issues in Leibniz's theory of time, which has remained too fragmentary and incomplete but nonetheless too necessary to a genuine understanding of Leibniz's theory of space for us to do without it. Needless to say, this all does not mean that the final result of such a reconstruction is perfectly consistent. On the contrary, in several places I have signaled the conceptual gaps of Leibniz's argumentation, as well as a few theories that – as far as I can understand (but I may be wrong) – can in no way be accommodated. Among them, I have already mentioned the problem of continuity and that of the characterization of similarity. Now, I would add the metaphysics of time. Not only, in fact, was the metaphysics of time almost absent as such in Leibniz, but it also risked being impossible to build – owing, as we will see (the subject is transversely treated through the various chapters), to a particular bend in Leibniz's theory of space.

On several occasions I had simply to choose between one reading and another, to favor a text or another one incompatible with the first. It is indeed according to such choices that an interpretation characterizes itself. In this sense, I think I have throughout favored a transcendental-phenomenological reading of Leibniz's philosophy, in many respects similar to the one that guided Cassirer more than a century ago. I think, in other words, that the core of Leibniz's late philosophy of space resides in his theory of a phenomenon and its knowledge. His main interest seems to me the rela-

tion of expression between monads and space, as well as the subjective but *a priori* determination of a phenomenon. In conclusion, the late Leibniz seems to me to be moving, if not towards Kant, at least towards a form of phenomenology. Thus, in dealing with the Principle of Indiscernibles, or the Principle of Individuation, or the general doctrine of intermonadic relations, or that of counterfactual truths, I have attempted to provide an account based on the relations between phenomena and things-in-themselves and the power of the representational faculties of a subject, which departs a little from (for example) the more common, purely logical (after Russell's fashion) accounts of the same principles and problems. I do not expect this reconstruction to be convincing in every point – one's wish always is that an interpretation, however rough it may be, will soon inspire a more refined one to come.